Concrete/Pictorial/Abstract

Maths Calculation Policy

Introduction

This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further consultation with teachers from Stanburn and material added. It is a working document and will be revised and amended as necessary. This policy is a statement of the aims, principles and strategies for teaching and learning of calculation strategies in Mathematics. It is designed to help teachers and staff at Stanburn Primary School to ensure that calculation is taught consistently across the school and to aid them in helping children who may need extra support or challenges. This policy is also designed to help parents, carers and other family members support children's learning by providing an explanation of the methods used in our school. The policy is set out in subjects: addition, subtraction, multiplication and division. Within each specific area, there is a progression of skills, knowledge and layout for written methods. The calculation strategies which will be used will reflect this ideology – moving from concrete to pictorial and then abstract recording leading to more formal written methods.

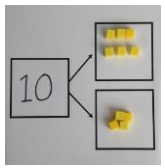
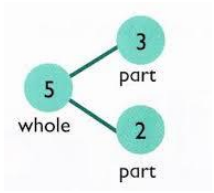
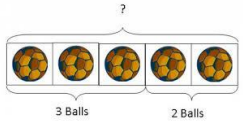
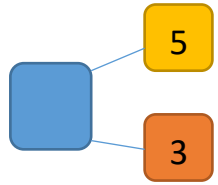
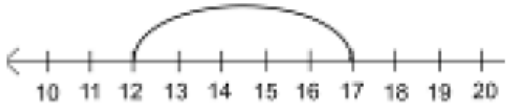
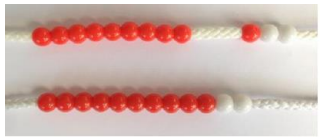
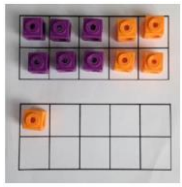
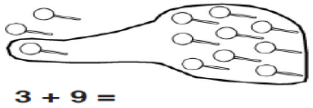
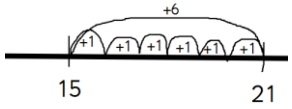
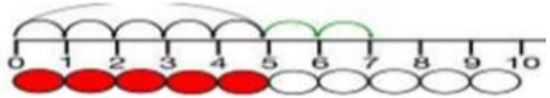
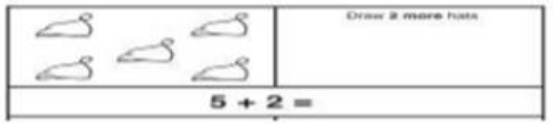
Concrete representation— a pupil is first introduced to an idea or skill by acting it out with real objects. This is a 'hands on' component using real objects and is a foundation for conceptual understanding.

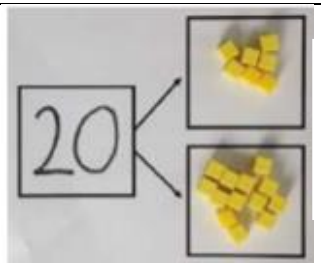
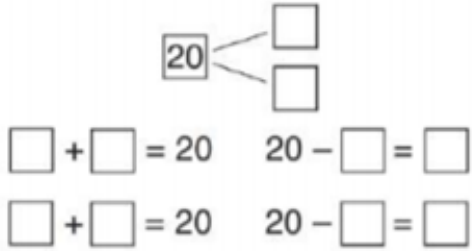
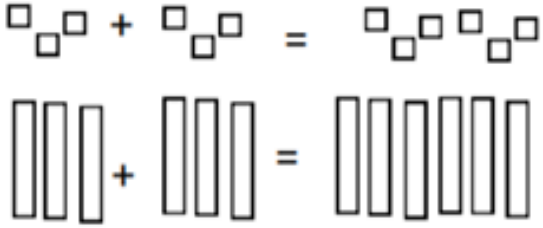
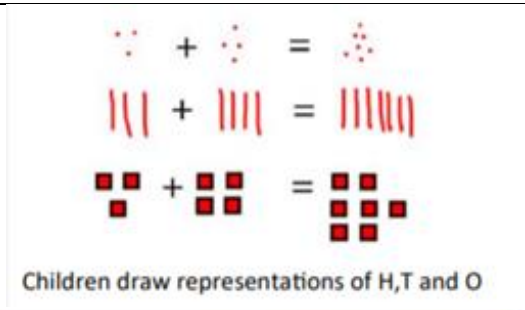
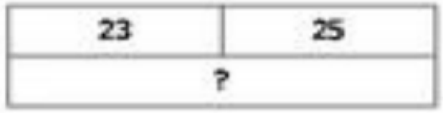
Pictorial representation – a pupil has sufficiently understood the 'hands on' experiences performed and can now relate them to representations, such as a diagram or picture of the problem.

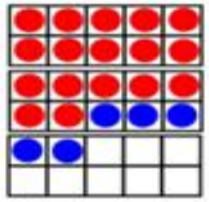
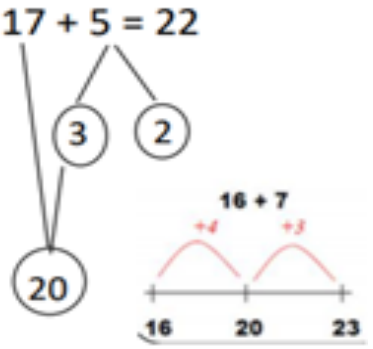
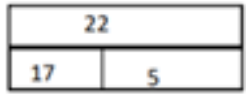
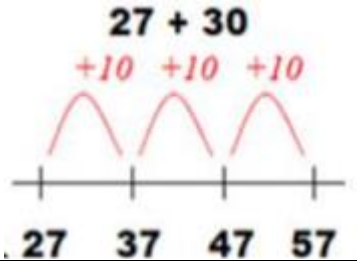
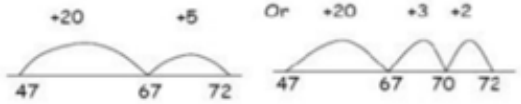
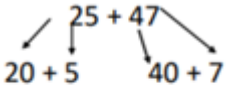
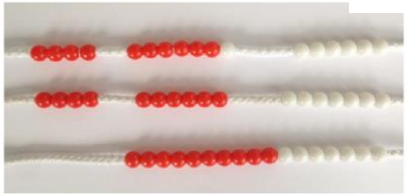
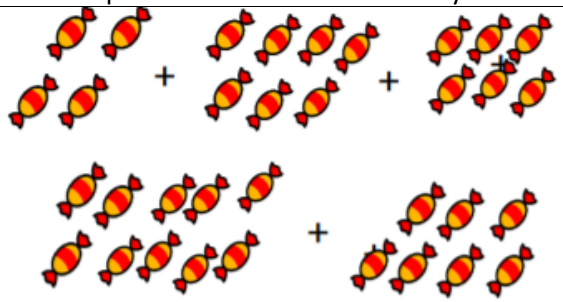
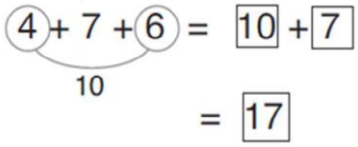
Abstract representation—a pupil is now capable of representing problems by using mathematical notation, for example 12 x 2 = 24. It is important that conceptual understanding, supported by the use of representation, is secure for all procedures. Reinforcement is achieved by going back and forth between these representations.

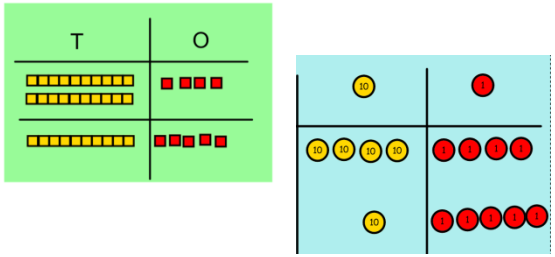
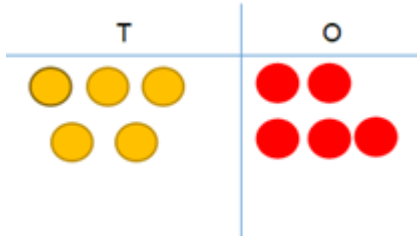
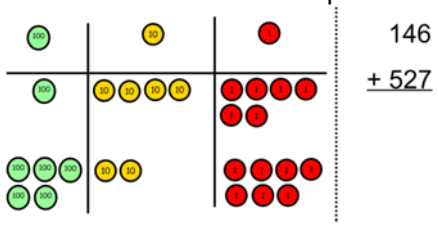
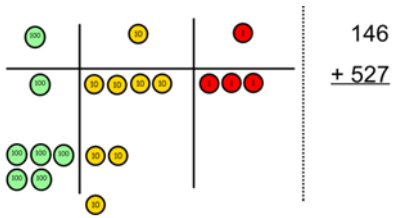
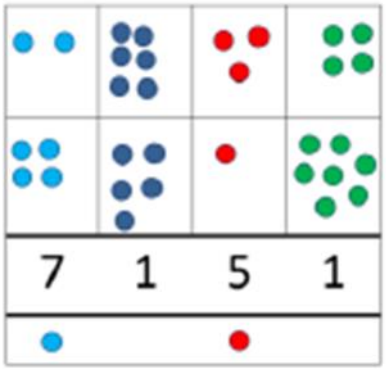
Mental methods and strategies will work in partnership with these methods. A variety of mental calculation methods and the recall of key facts will be taught in school and tested regularly. The progression of mental methods and expectations will comply with the National Curriculum Statements from July 2014. The basis of our maths calculation policy is that written methods are complementary to mental methods and should not be seen as separate from them. Children should use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

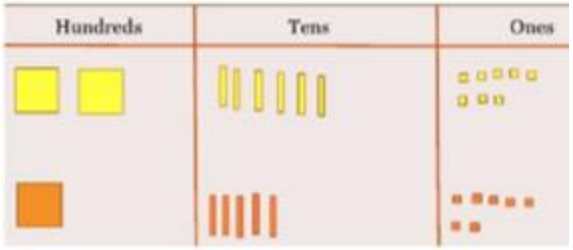
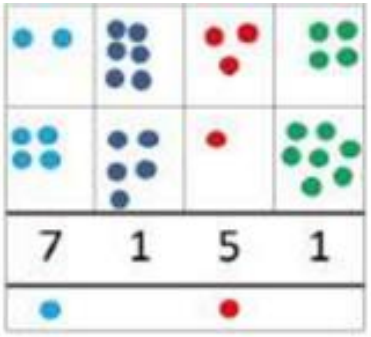
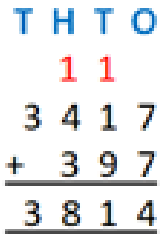
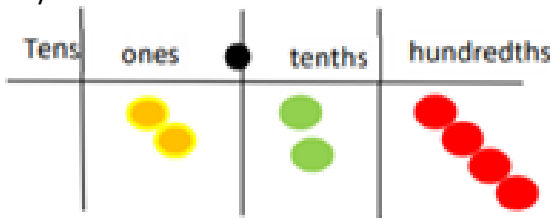
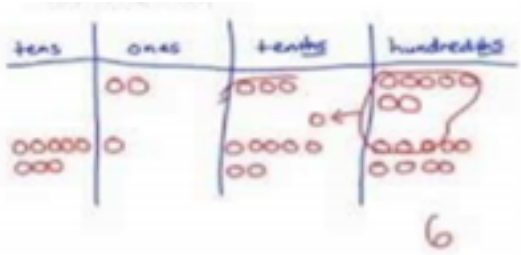
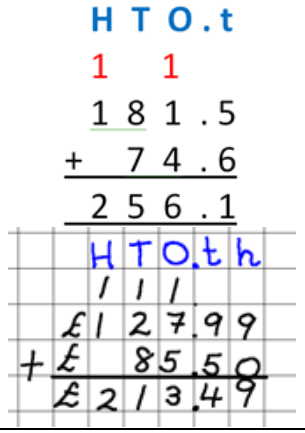
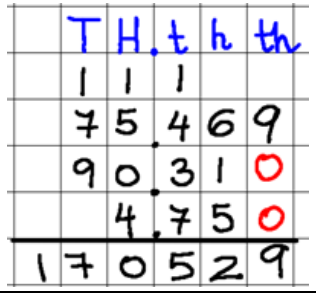
MathsHUBS

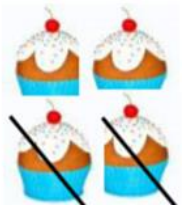
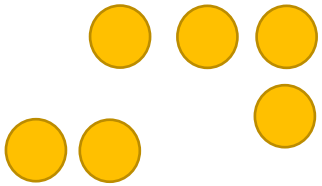
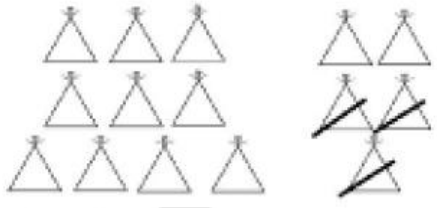
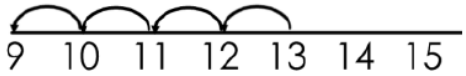
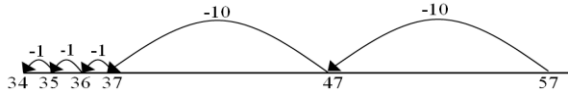
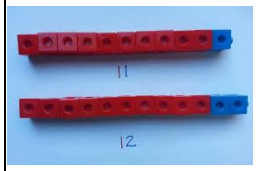
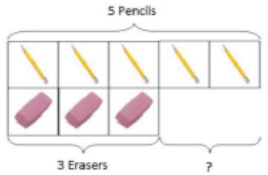
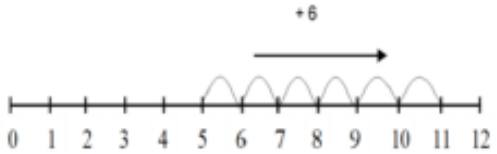
	Objective and Strategies	Concrete	Pictorial	Abstract
Y1 ADDITION	Combining two parts to make a whole: part-whole model	   Use part, part, whole model. Use cubes to add two numbers together as a group or in a bar. 	    Use pictures to add two numbers together as a group or in a bar.	4 + 3 = 7  10= 6 + 4 Use the part-part whole diagram as shown above to move into the abstract.
	Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
	Regrouping to make 10. *This is an essential skill for column addition later*	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10. Use ten frames.	 Use pictures or a number line. Regroup or partition the smaller number to make 10. 	7 + 4 = 11 If I am at seven, how many more do I need to make 10? How many more do I add on now?
	Represent & use number bonds and related subtraction facts within 20	 2 more than 5	 	The emphasis is on the language: *'1 more than 5 is equal to 6.'* *'2 more than 5 is 7.'* *'8 is 3 more than 5.'*

	Objective and Strategies	Concrete	Pictorial	Abstract
Y2 ADDITION	Adding multiples of 10	50 = 30 + 20  Model using dienes and bead strings.	 3 tens + 5 tens = __ tens 30 + 50 = __ Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + __ = 60
	Use known number facts Part part whole	 Children explore ways of making numbers within 20.	 20 □ + □ = 20 20 − □ = □ □ + □ = 20 20 − □ = □	□ + 1 = 16 16 − 1 = □ 1 + □ = 16 16 − □ = 1
	Use known facts		 Children draw representations of H,T and O	3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700
	Bar model	 3 + 4 = 7	 7 + 3 = 10	 23 25 ? 23 + 25 = 48

Y2 ADDITION	Add a 2 digit number and ones	 17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	Use part part whole and number line to model. 	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17 
	Add a 2 digit number and tens	 25 + 10 = 35 Explore that the ones digit does not change		27 + 10 = 37 27 + 20 = 47 27 + __ = 57
	Add two 2 digit numbers	 Model using dienes , place value counters and numicon	 Use the number line and bridge 10 using the part whole model if necessary.	 20 + 40 = 60 5+ 7 =12 60 + 12 = 72
	Adding three single digits	4 + 7 + 6 = 17 Put 4 and 6 together to make dd on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.

	Objective and Strategies	Concrete	Pictorial	Abstract
Y3 ADDITION	Column method- no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. 	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 	H T O 2 2 3 + 1 1 4 3 3 7 Add the ones first, then tens, then hundreds.
	Column method with regrouping	Make both numbers on the place value grid.  Add up the units and exchange 10 ones for one 10  Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding. 	Start by partitioning the numbers before moving on to clearly show the exchange **above** the addition. 20 + 5 40 + 8 60 + 13 = 73 H T O 1 3 4 7 + 1 7 2 5 1 9

	Objective and Strategies	Concrete	Pictorial	Abstract
Y4 - Y6 ADDITION	Y4 – Add numbers with up to 4 digits	Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	Draw representations using the place value grid 	Continue from previous work to carry hundreds as well as tens. Relate to money and measures. 
	Y5 – Add numbers with more than 4 digits Add decimals with 2 decimal places, including money	As year 4  Introduce decimal place value counters and model exchange for addition.	2.37 + 81.79 	
	Y6 – Add several numbers of increasing complexity. Include adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5	Put the zeros for the place holder Use known facts when adding more than two digits – eg number bonds to 10 or doubles or near doubles. 

	Objective and Strategies	Concrete	Pictorial	Abstract
Y1 SUBTRACTION	Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away. 4 – 2 = 2  6 – 2 = 4 	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	18 -3= 15 8 – 2 = 6
	Counting back	13 – 4  Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  9 10 11 12 13 14 15 Start at the bigger number and count back the smaller number showing the jumps on the number line.  -10 -10 -1 -1 -1 34 35 36 37 47 57 This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.
	Find the difference	 Compare amounts and objects to find the difference. Use cubes to build towers or make bars to find the difference Use basic bar models with items to find the difference.  5 Pencils 3 Erasers ?	Count on to find the difference.  +6 0 1 2 3 4 5 6 7 8 9 10 11 12	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.

<table>
<tr>
<td rowspan="3">Y1

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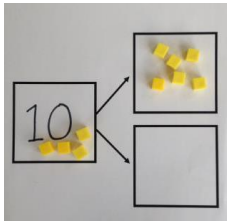
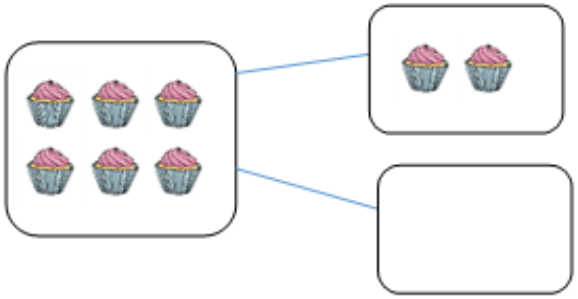
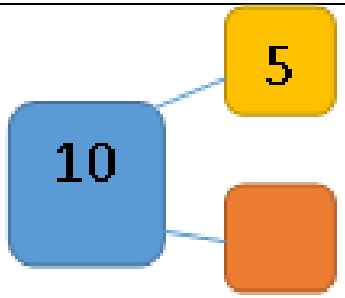
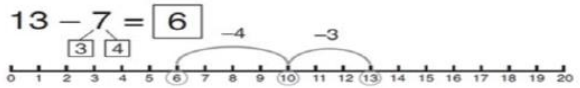
N</td>
<td>Represent and use number bonds and related subtraction facts within 20

Part part whole</td>
<td> 

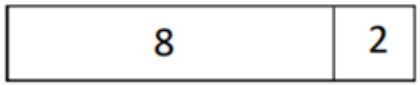
Link this to addition- use the part whole model to help explain the inverse between addition and subtraction.

10 - 6 =
If 10 is the whole and 6 is one of the parts, what is the other part?</td>
<td>Use a pictorial representation of objects to show the part part whole model.
  </td>
<td> 

Move to using numbers within the part whole model.</td>
</tr>
<tr>
<td>Make 10</td>
<td>14 – 5 =
 
Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5.</td>
<td> 

Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</td>
<td>16 – 8 =

How many do we take off to reach the next 10?

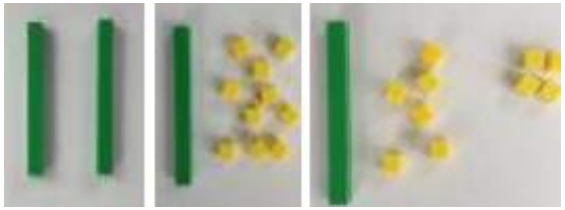
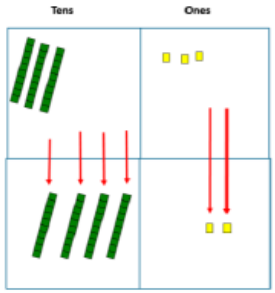
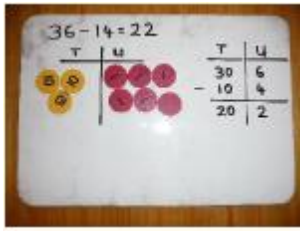
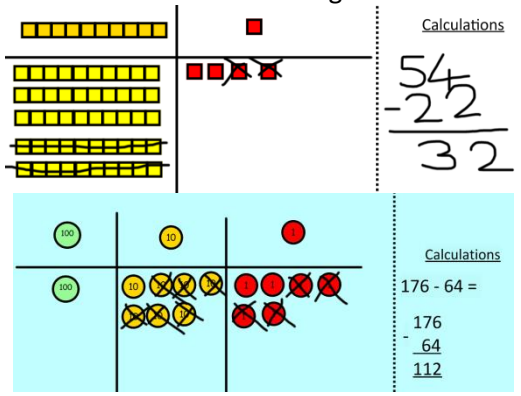
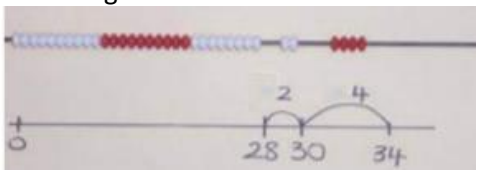
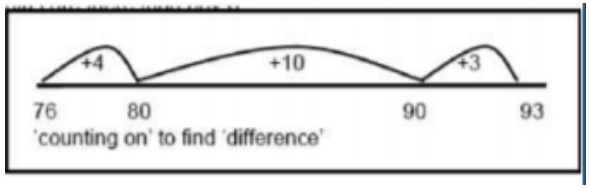
How many do we have left to take off?</td>
</tr>
<tr>
<td>Bar model</td>
<td>5 – 2 =
  </td>
<td>  </td>
<td> 

10 = 8 + 2

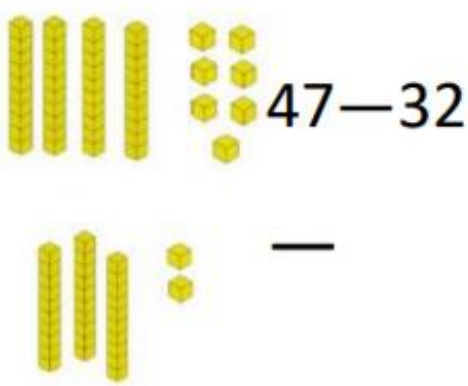
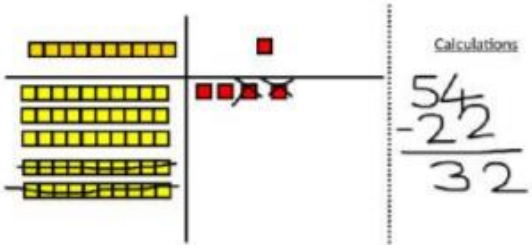
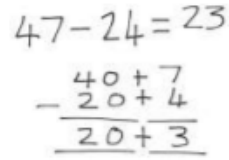
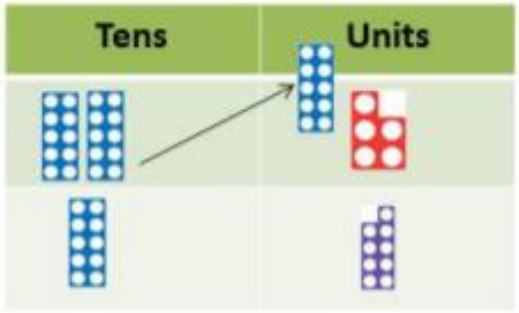
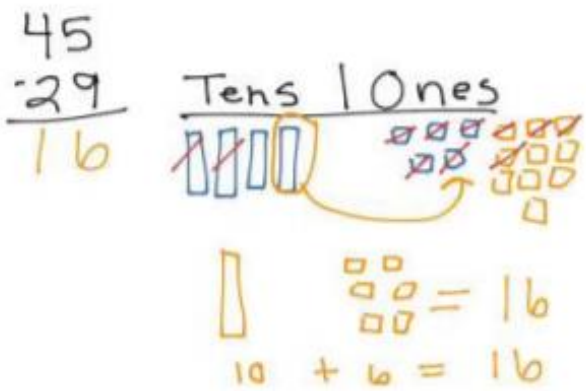
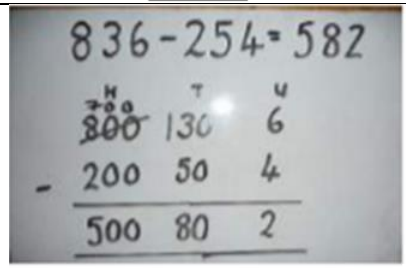
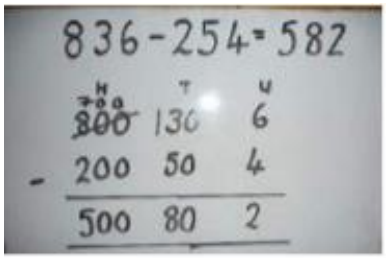
10 = 2 + 8

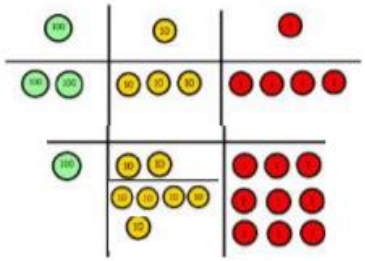
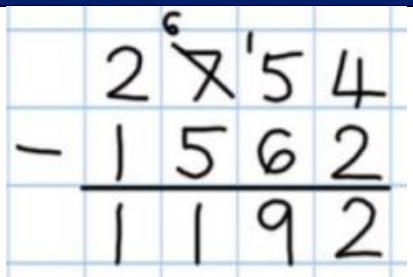
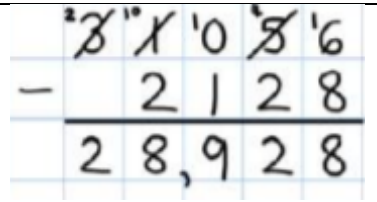
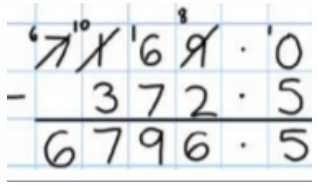
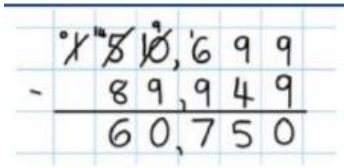
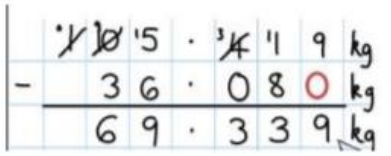
10 – 2 = 8

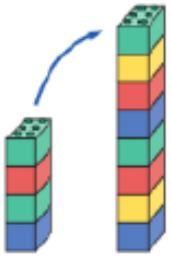
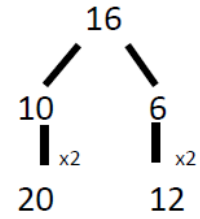
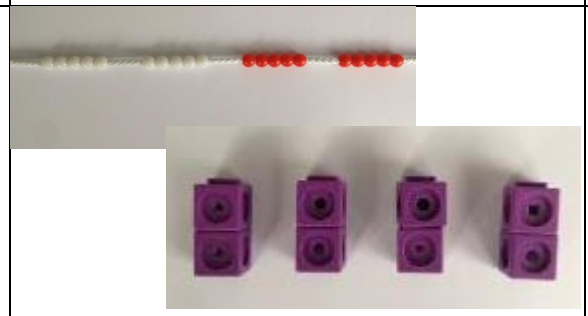
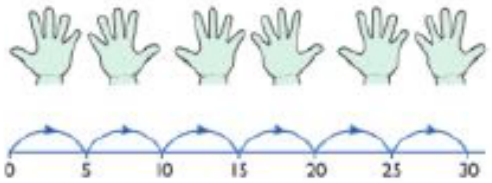
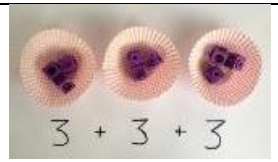
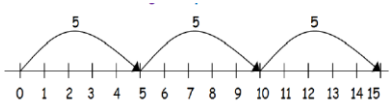
10 -8 = 2</td>
</tr>
</table>

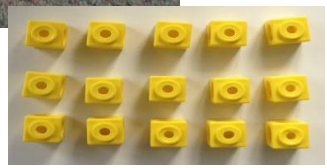
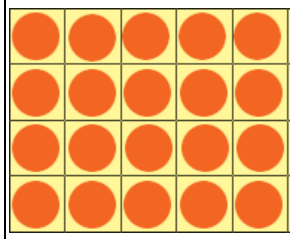
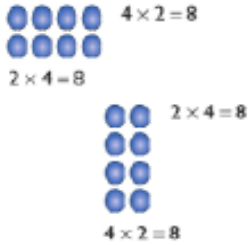
	Objective and Strategies	Concrete	Pictorial	Abstract
Y2 SUBTRACTION	Regroup a ten into ten ones	Use a place value chart to show how to change a ten into ten ones, use the term 'take and make'. 	20 – 4 = 	20 – 4 = 16
	Partitioning to subtract without regrouping	 Use Base 10 to make the bigger number then take the smaller number away. 36 – 14 = Show how you partition numbers to subtract. Again make the larger number first. 	Draw the Base 10 or place value counters alongside the written calculation to help to show working. 	43 – 21 = 22
	Make ten strategy Progression should be crossing one ten, crossing more than one ten, crossing the hundred.	34 – 28 = Use a bead bar or bead strings to model counting to the next ten and the rest. 	 Use a number line to count on to next ten and then the rest.	93 – 76 = 17
	Objective and Strategies	Concrete	Pictorial	Abstract

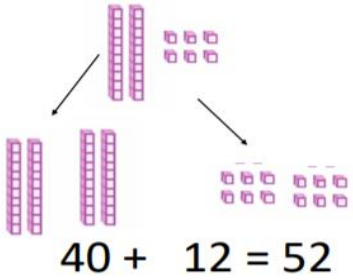
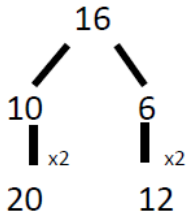
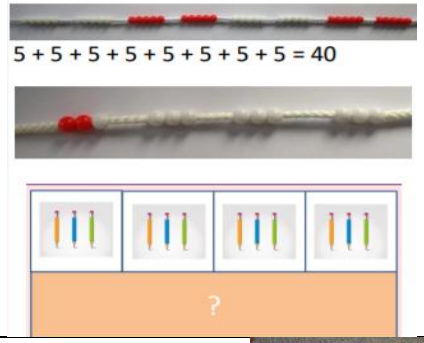
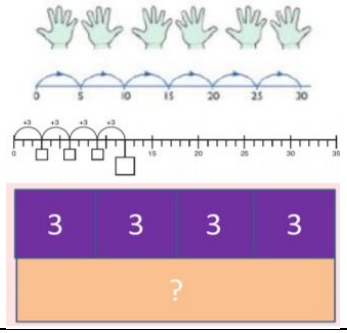
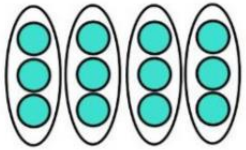
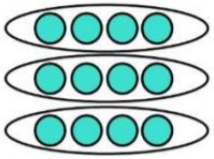
Y3 SUBTRACTION

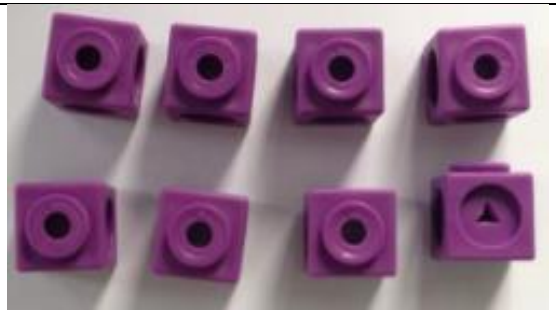
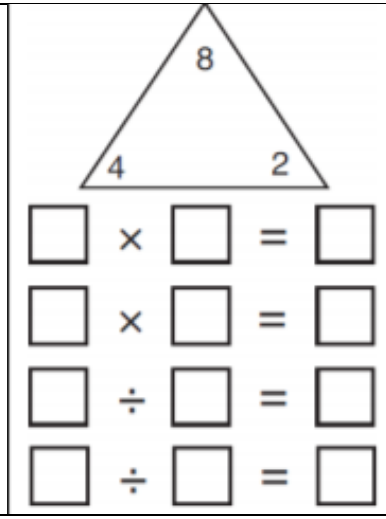
Column subtraction without regrouping	 Use base 10 or Numicon to model	 Draw representations to support understanding	 Intermediate step may be needed to lead to clear subtraction understanding T O 3 2 - 1 2 2 0
Column subtraction with regrouping	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters 	Children may draw base ten or place value counters and cross off. 	 Begin by partitioning into place value columns.  Then move to formal method.

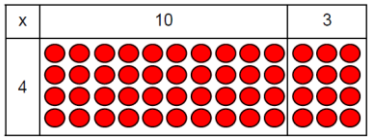
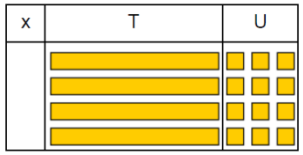
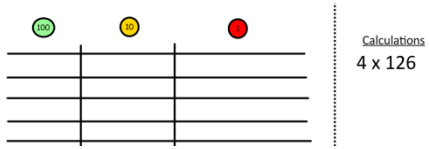
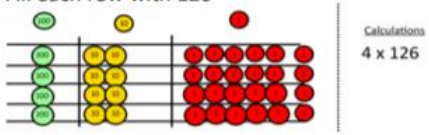
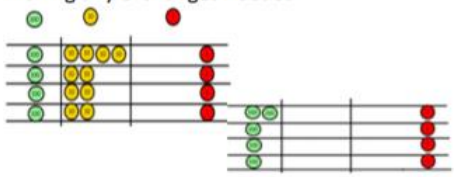
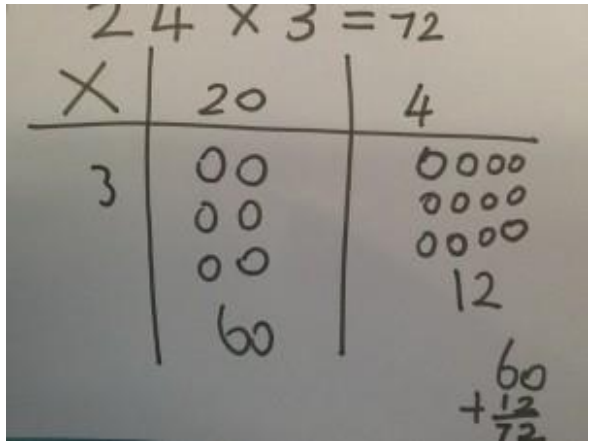
	Objective and Strategies	Concrete	Pictorial	Abstract
Y4 - Y6 SUBTRACTION	Subtracting tens and ones Y4 – Subtracting with up to 4 digits. Introduce decimal subtraction through the context of money	234 - 179  Model the process of exchanging using Numicon, base ten and then move to place value counters.	Children will draw the place value counters and show their exchange – see Y3	 Use the phrase 'take and make' for exchange.
	Y5 – Subtract with at least 4 digits, including money and measure. Subtract with decimal values, including mixtures of integers and decimals and align the decimals.	As Y4	Children will draw the place value counters and show their exchange – see Y3	 Use zeros for place holders. 
	Y6 – Subtract with increasingly large and more complex numbers and decimal values			 

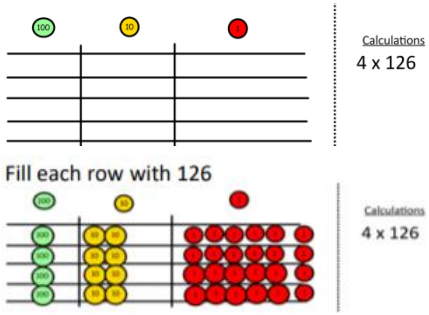
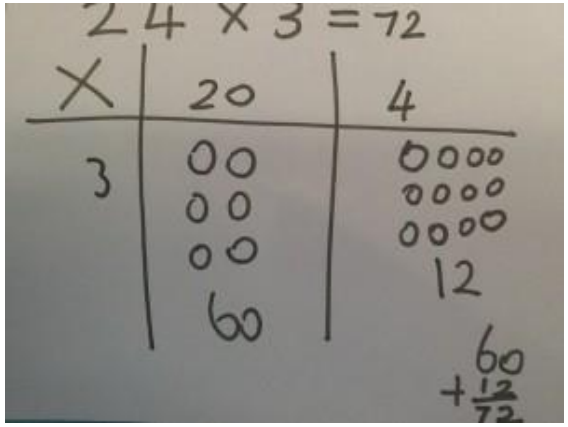
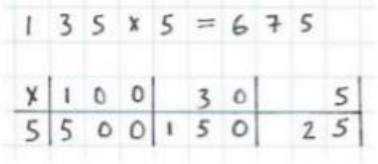
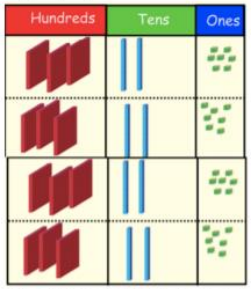
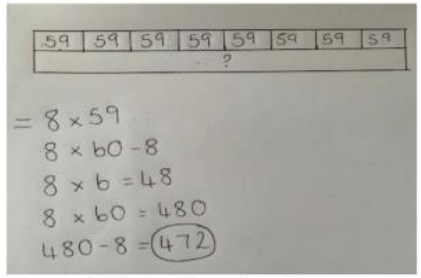
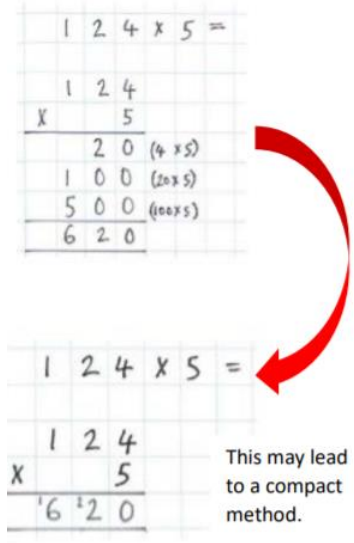
	Objective and Strategies	Concrete	Pictorial	Abstract
Y1 MULTIPLICATION	Doubling	Use practical activities to show how to double a number.  double 4 is 8 4 × 2 = 8	Draw pictures to show how to double a number. Double 4 is 8 	 Partition a number and then double each part before recombining it back together.
	Counting in multiples	 Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
	Repeated addition	   Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6 Use pictorial representations, including number lines, to solve problems.  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10

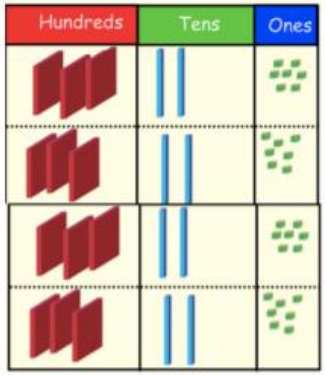
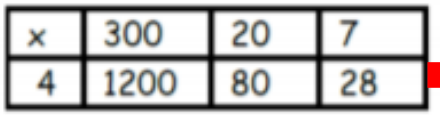
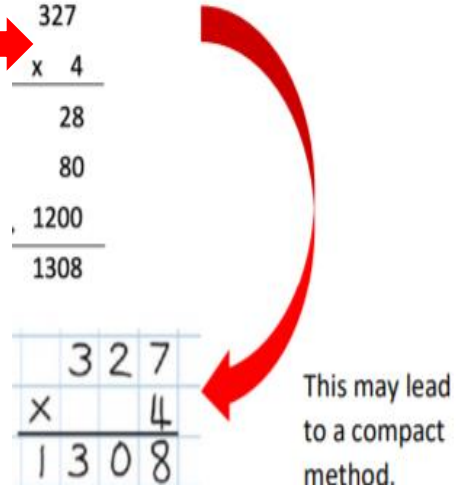
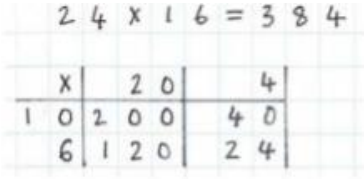
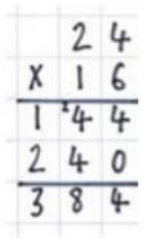
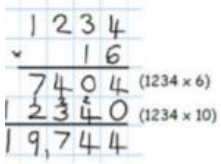
	Understanding arrays	Create arrays using counters/cubes to show multiplication sentences.  	Draw arrays in different rotations to find commutative multiplication sentences.   Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

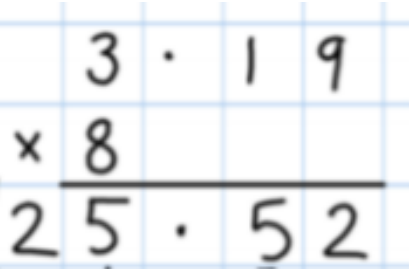
	Objective and Strategies	Concrete	Pictorial	Abstract
Y2 MULTIPLICATION	Doubling	Model doubling using dienes and PV counters.  	Draw pictures and representations to show how to double numbers	 Partition a number and then double each part before recombining it back together.
	Counting in multiples of 2, 3, 5 and 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.  	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$
	Multiplication is commutative	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.	Use representations of arrays to show different calculations and explore commutativity.  	12 = 3 × 4 12 = 4 × 3 Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15

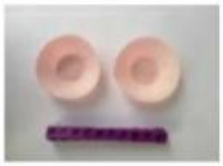
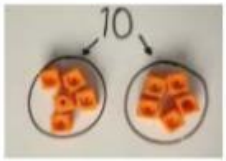
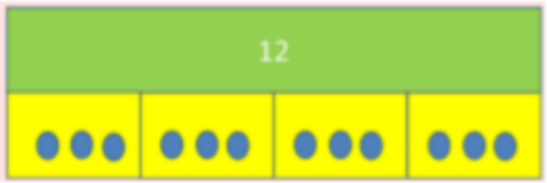
	Using the inverse This should be taught alongside division, so pupils learn how they work alongside each other.				2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8 ÷ 2 Show all 8 related fact family sentences.

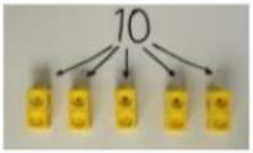
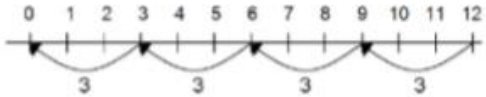
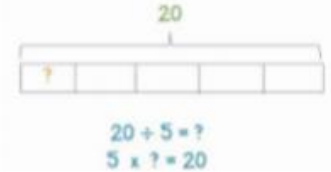
	Objective and Strategies	Concrete	Pictorial	Abstract
Y3 MULTIPLICATION	Multiplying two digit number by a one digit number Grid method progressing to the formal method. Solving problems including missing number problems, integer scaling problems.	Show the link with arrays to first introduce the grid method.  4 rows of 10 4 rows of 3 Move on to using Base 10 to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.  Fill each row with 126  Add up each column, starting with the ones making any exchanges needed  Then you have your answer.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | ×	30	5
---	---	---
7	210	35 | **210 + 35 = 245** Moving forward, multiply by a 2 digit number showing the different rows within the grid method. |	10	8
---	---	---
10	100	80
3	30	24 | | X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

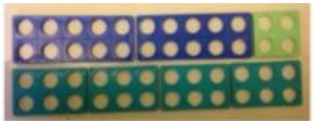
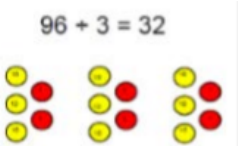
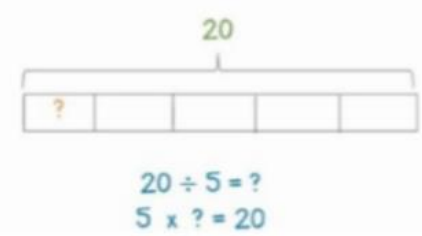
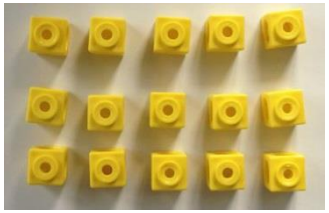
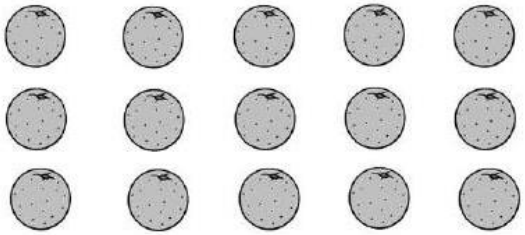
	Objective and Strategies	Concrete	Pictorial	Abstract
Y4 MULTIPLICATION	Grid method recap from year 3 for 2 digits x 1 digit Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)		Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below. 	HTO x O  Children add up each column to find the answer.
CALCULATION	Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication. This is initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside Model | Calculation	This grid method may be used to show how this relates to a formal written method. | x | 100 | 20 | 4 | | 5 | 500 | 100 | 20 |  Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.	 This may lead to a compact method.

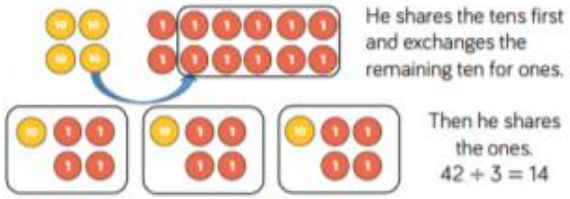
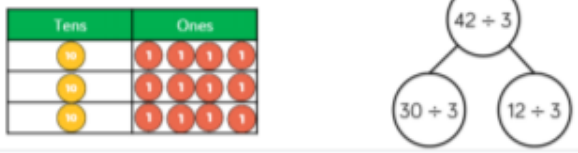
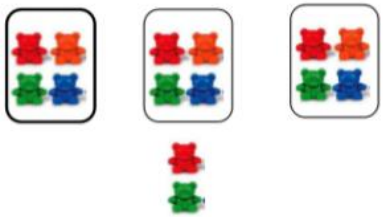
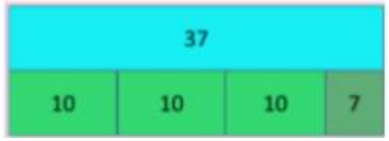
	Objective and Strategies	Concrete	Pictorial	Abstract
Y5 MULTIPLICATION	Multiply numbers up to 4-digits by a one-digit number using the formal written method, including long multiplication for 2-digit numbers Column multiplication for 3 and 4 digits x 1 digit	Children can continue to be supported by place value counters at the stage of multiplication. This is initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside **Model** / **Calculation**	 	 This may lead to a compact method.
	Column multiplication (long multiplication)	Manipulatives may still be used with the corresponding long multiplication modelled alongside. **Model** / **Calculation**	Moving forward, multiply by a 2 digit number showing the different rows within the grid method.  	 24 x 6 on the first row. (6 x 4 = 24, carrying the 2 for the 20, then 6 x 2) 24 x 10 on the second row. Show multiplying by 10 by putting zero in the units first. 

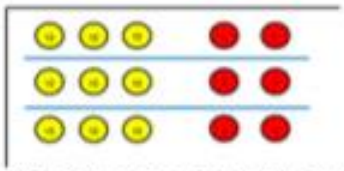
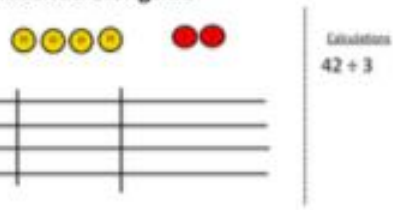
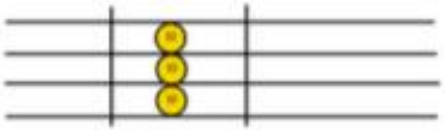
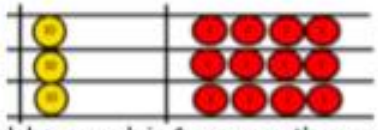
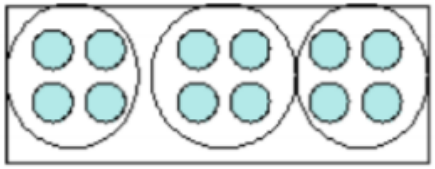
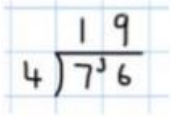
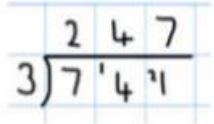
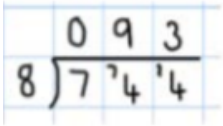
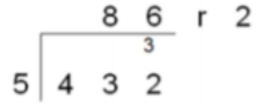
	Objective and Strategies	Concrete	Pictorial	Abstract
Y6 MULTIPLICATION	Multiply decimal up to 2 decimal place by a single digit.			Remind children that the single digit belongs in the ones column. Line up the decimals point in the question and in the answer 

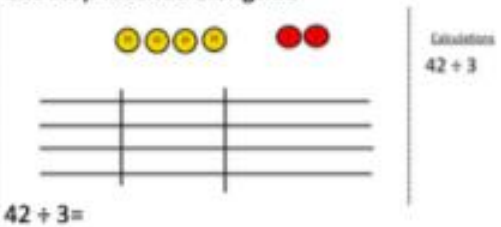
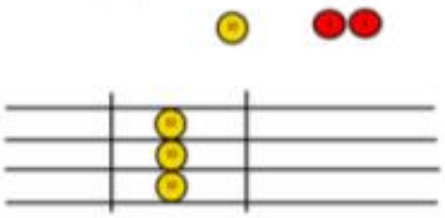
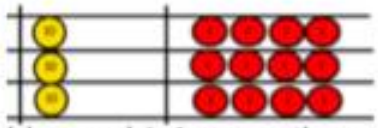
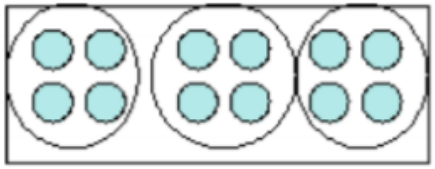
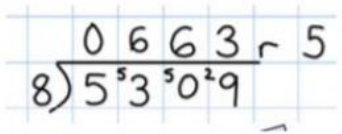
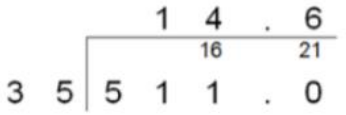
	Objective and Strategies	Concrete	Pictorial	Abstract
Y1 DIVISION	Division as sharing (sharing objects into groups)	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  $8 \div 2 = 4$ Children use bar modelling to show and support understanding. 	Share 9 buns between three people $9 \div 3 = 3$

	Objective and Strategies	Concrete	Pictorial	Abstract
Y2 D I V I S I O N	Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  96 ÷ 3 = 32   	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you ar dividing by and work out how many would be within each group.  	**28 ÷ 7 = 4** **Divide 28 into 7 groups. How many are in each group?**

	Objective and Strategies	Concrete	Pictorial	Abstract
Y3 DIVISION	Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups 6 = 4 	Continue to use bar modelling to aid solving division problems. 	How many groups of 6 in 24? 24 ÷ 6 = 4
	Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 15 ÷ 5 = 3 5 x 3 = 15 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4
	Divide a two digit number by a one digit number by partitioning into tens and ones	Eva uses a place value grid and the part whole model to solve 66 ÷ 3 	See part whole number	

Y3 DIVISION	Divide numbers that involve exchanging between the tens and ones. The answers do not have remainders.	Ron uses place value counters to divide 42 into three equal groups.  	Annie uses a similar method to divide 42 by 3  Children may use pictorial representations for the place value counters, alongside the part whole model. Children use their times tables to partition the number into multiples of the divisor.	96 ÷ 8 96 ÷ 4 96 ÷ 3 96 ÷ 6 Compare the statements using <, > or = 48 ÷ 4 ◯ 36 ÷ 3 52 ÷ 4 ◯ 42 ÷ 3 60 ÷ 3 ◯ 60 ÷ 4
	Division with a remainder	**14 ÷ 3 =** Divide objects between groups and see how much is left over.  Moving on to: Use place value counters to work out 94 ÷ 4 Did you need to exchange any tens for ones? Is there a remainder?  	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.  	Complete written division and show the remainder using r 

	Objective and Strategies	Concrete	Pictorial	Abstract
Y4 DIVISION	Divide up to 3 digit number by a 1 digit number Short Division	96 ÷ 3 Tens Units 3 2 3  Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.   We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Student can continue to use drawn diagrams with dots or circles to help divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with a divisor that divides equally with no remainders   Children should be aware that the 0 is used to keep the place value, if the number is not divisible.  Move on to division with remainders 

	Objective and Strategies	Concrete	Pictorial	Abstract
Y5 DIVISION	Divide at least 4 digit numbers by 1 digit. Interpret remainders appropriately for the context. Short Division	96 ÷ 3 Tens Units 3 2 3  Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Student can continue to use drawn diagrams with dots or circles to help divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	 Finally move into decimal places to divide the total accurately. 

	Objective and Strategies	Concrete	Pictorial	Abstract
Y6 D I V I S I O N	Children will use long division to divide numbers with up to 4 digits by 2 digit numbers. Long Division			046.2 15)693.0 - 60 93 - 90 3.0 - 3.0 0